Lecture 1

Course Overview

Overview of Complexity Theory

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Algorithms vs Complexity Theory:

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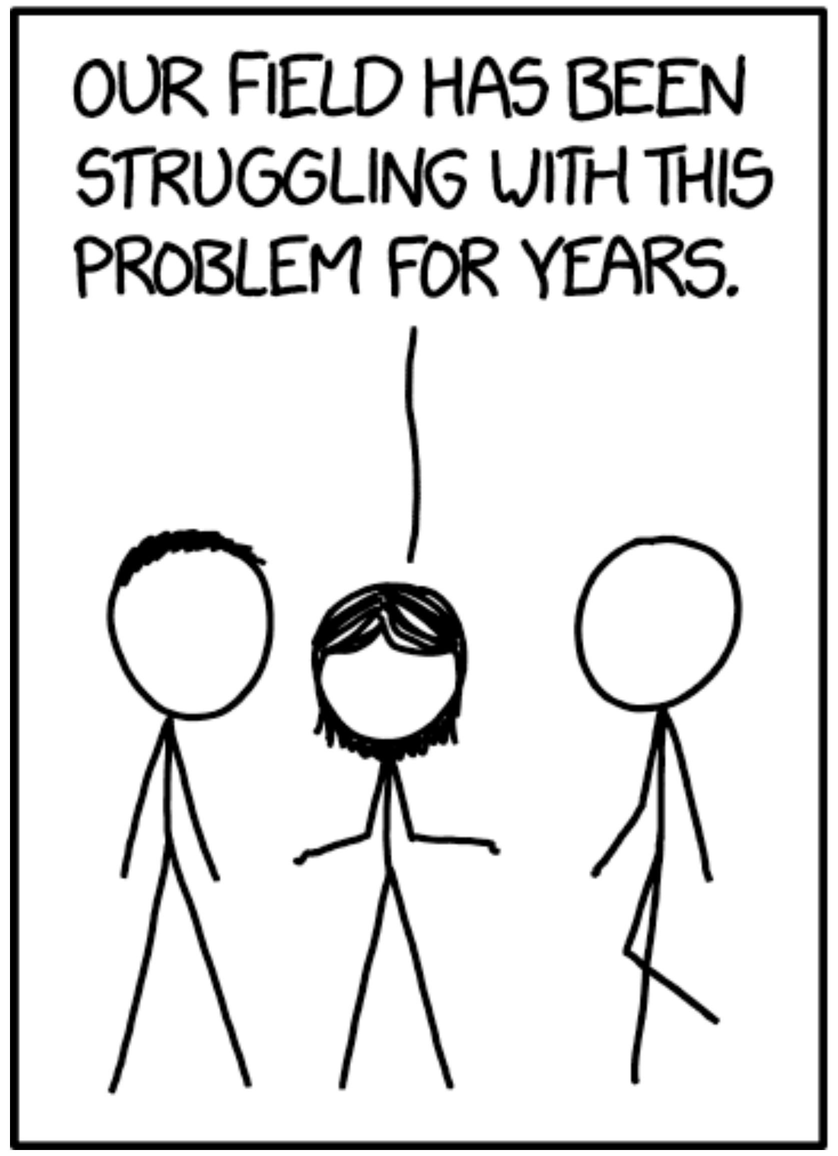

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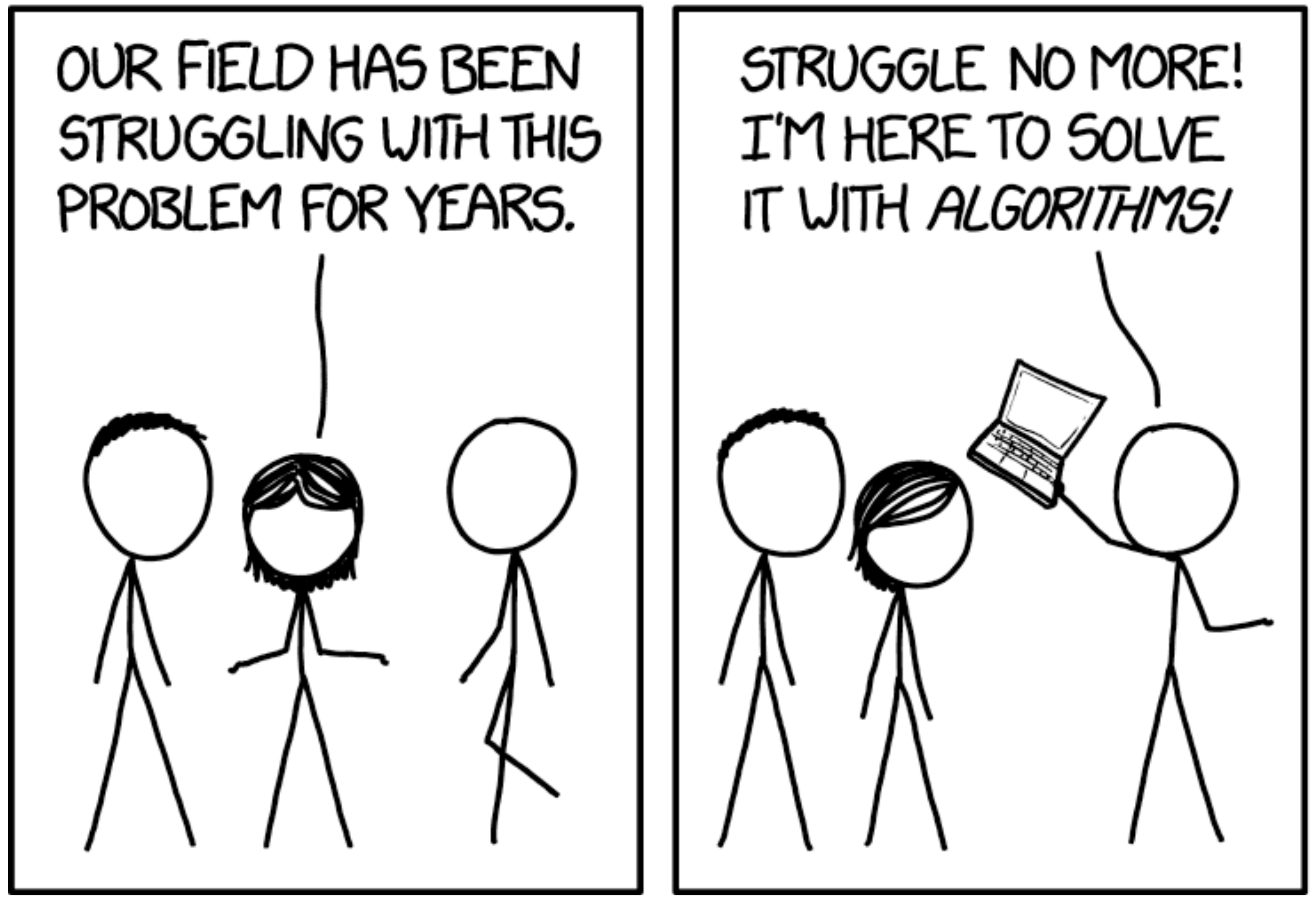

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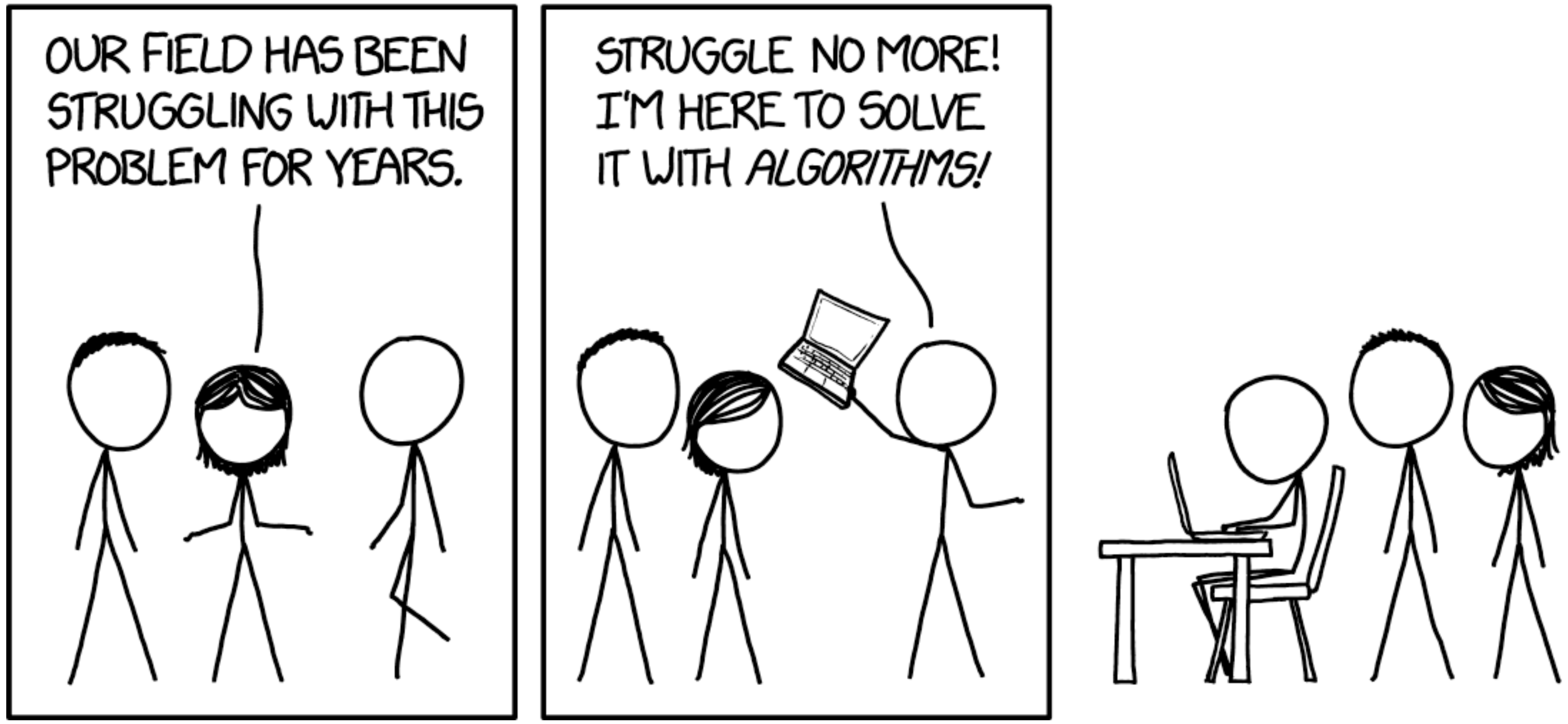

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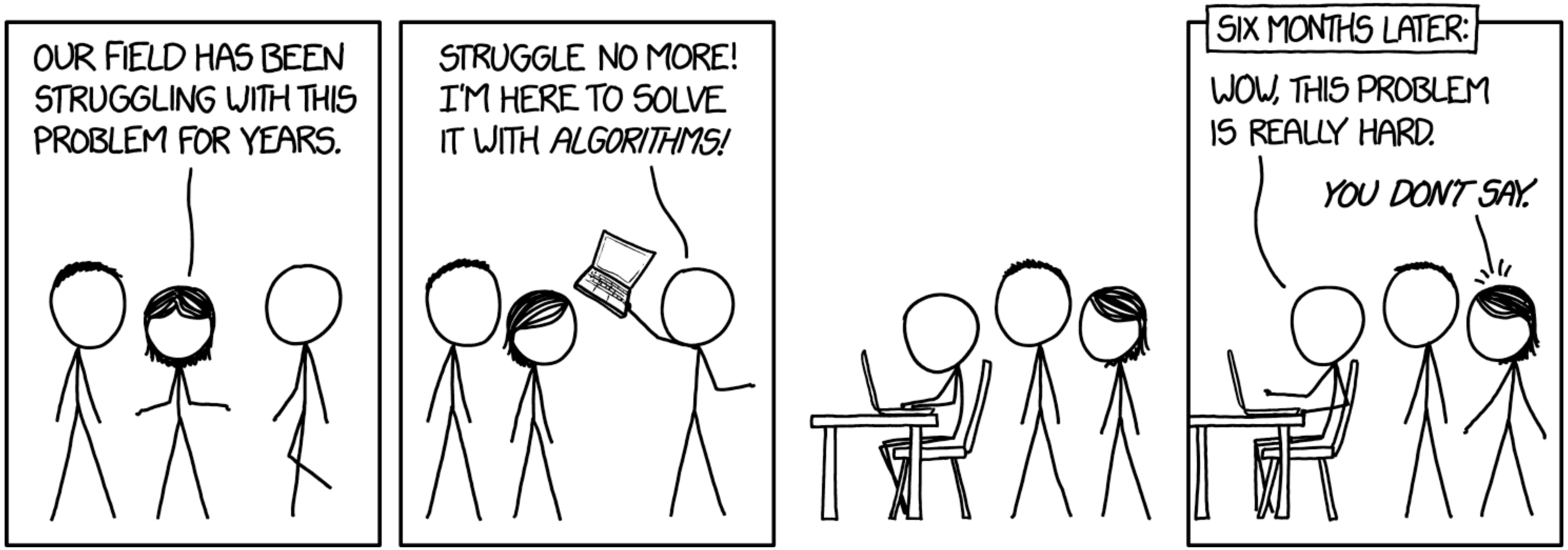

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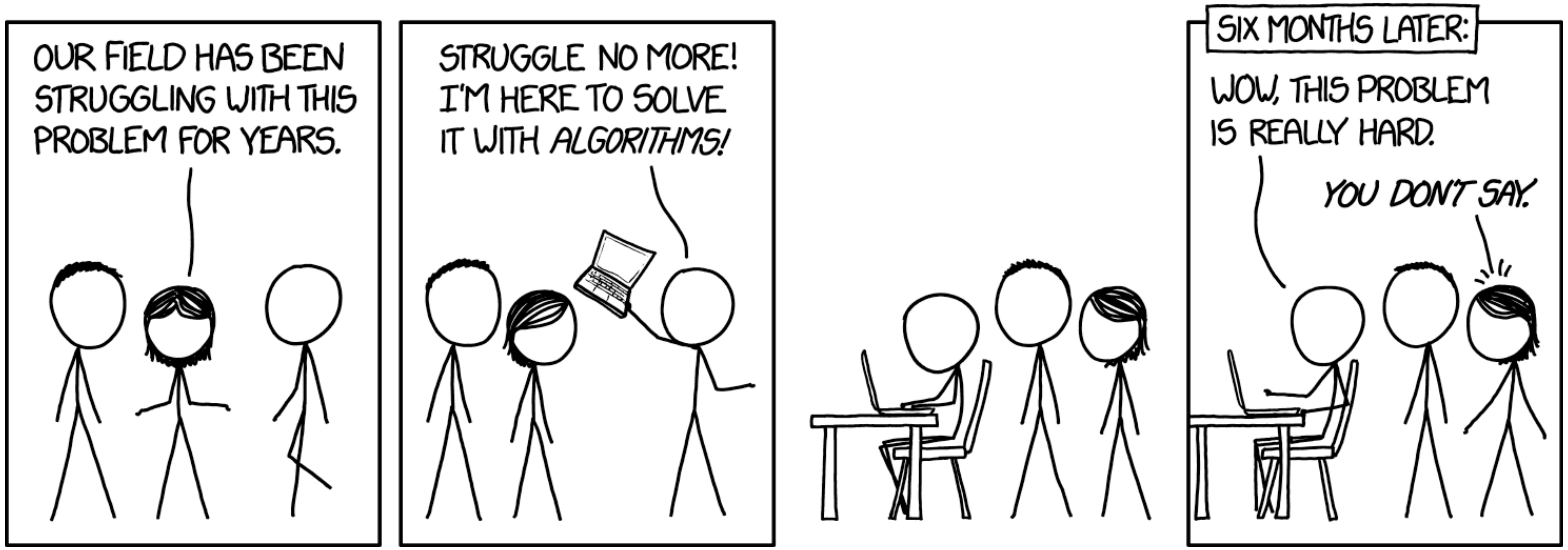

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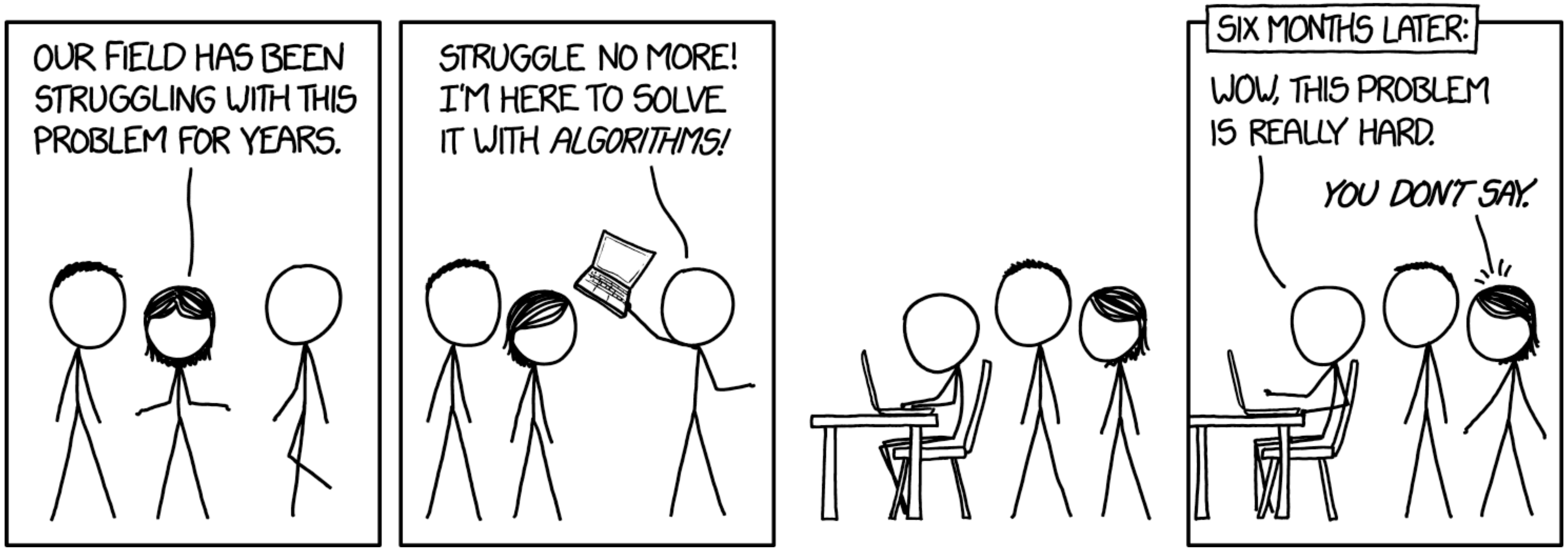

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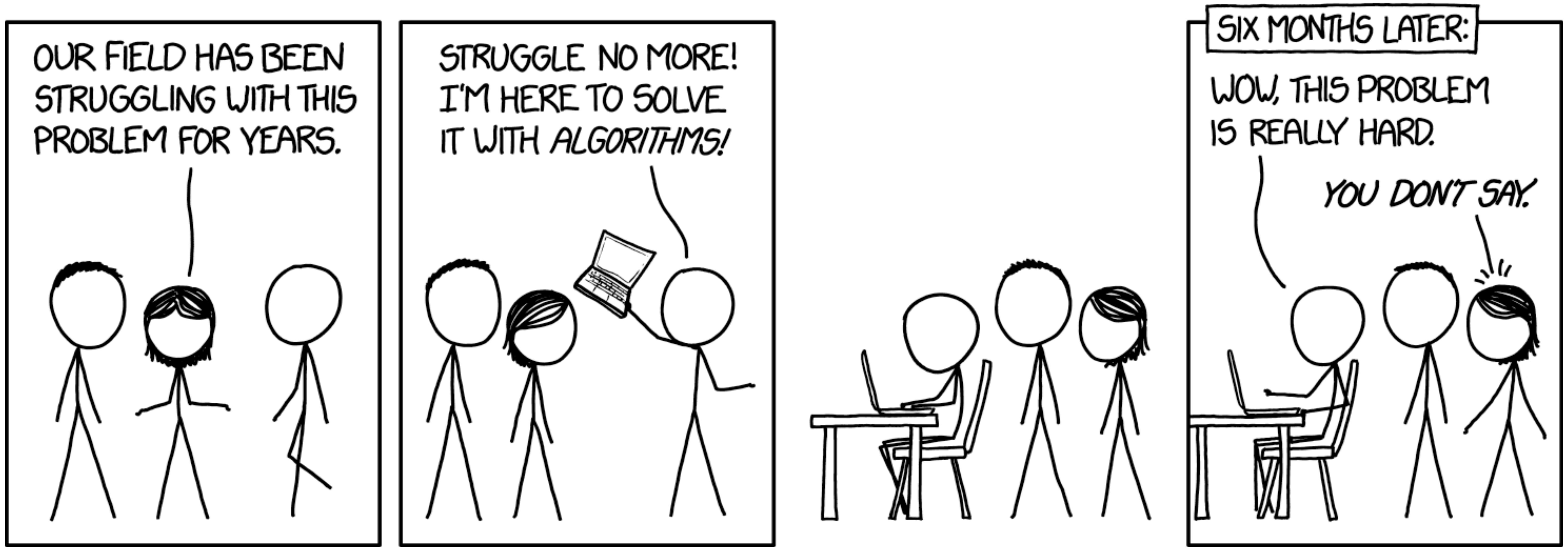

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Haven't been very successful

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- Are there problems solvable in $O(n^3)$ time that are not solvable in $O(n)$ time?

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  Easily verifiable solutions to *EXACT-INDSET* seem to not exist. (*EXACT-INDSET* $\in \Sigma_2^p$)

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Course Site: http://home.iitj.ac.in/~vimalraj/courses/ct/csl7140.html